

- b) First rewrite the equation with 0 on the right-hand side and the left-hand side in order of descending exponents:

$$\begin{aligned} 3x - 6x^2 &= -9 && \text{Original equation} \\ -6x^2 + 3x + 9 &= 0 && \text{Add 9 to each side.} \\ 2x^2 - x - 3 &= 0 && \text{Divide each side by } -3. \\ (2x - 3)(x + 1) &= 0 && \text{Factor.} \\ 2x - 3 = 0 &\quad \text{or} \quad x + 1 = 0 && \text{Zero factor property} \\ x = \frac{3}{2} &\quad \text{or} \quad x = -1 && \text{Solve for } x. \end{aligned}$$

The solution set is $\left\{-1, \frac{3}{2}\right\}$. Check each solution in the original equation. ■

CAUTION If we divide each side of $10x^2 = 5x$ by $5x$, we get $2x = 1$, or $x = \frac{1}{2}$. We do not get $x = 0$. By dividing by $5x$ we have lost one of the factors and one of the solutions.

In the next example there are more than two factors, but we can still write an equivalent equation by setting each factor equal to 0.

EXAMPLE 3

Solving a cubic equation by factoring

Solve $2x^3 - 3x^2 - 8x + 12 = 0$.

Solution

First notice that the first two terms have the common factor x^2 and the last two terms have the common factor -4 .

$$\begin{aligned} x^2(2x - 3) - 4(2x - 3) &= 0 && \text{Factor by grouping.} \\ (x^2 - 4)(2x - 3) &= 0 && \text{Factor out } 2x - 3. \\ (x - 2)(x + 2)(2x - 3) &= 0 && \text{Factor completely.} \\ x - 2 = 0 &\quad \text{or} \quad x + 2 = 0 &\quad \text{or} \quad 2x - 3 = 0 && \text{Set each factor equal to 0.} \\ x = 2 &\quad \text{or} \quad x = -2 &\quad \text{or} \quad x = \frac{3}{2} \end{aligned}$$

The solution set is $\left\{-2, \frac{3}{2}, 2\right\}$. Check each solution in the original equation. ■

The equation in the next example involves absolute value.

EXAMPLE 4

Solving an absolute value equation by factoring

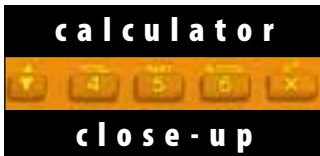
Solve $|x^2 - 2x - 16| = 8$.

Solution

First write an equivalent compound equation without absolute value:

$$\begin{aligned} x^2 - 2x - 16 &= 8 &\quad \text{or} \quad x^2 - 2x - 16 &= -8 \\ x^2 - 2x - 24 &= 0 &\quad \text{or} \quad x^2 - 2x - 8 &= 0 \\ (x - 6)(x + 4) &= 0 &\quad \text{or} \quad (x - 4)(x + 2) &= 0 \\ x - 6 = 0 &\quad \text{or} \quad x + 4 = 0 &\quad \text{or} \quad x - 4 = 0 &\quad \text{or} \quad x + 2 = 0 \\ x = 6 &\quad \text{or} \quad x = -4 &\quad \text{or} \quad x = 4 &\quad \text{or} \quad x = -2 \end{aligned}$$

The solution set is $\{-2, -4, 4, 6\}$. Check each solution. ■



To check, use $Y=$ to enter $y_1 = 2x^3 - 3x^2 - 8x + 12$. Then use the variables feature (VARS) to find $y_1(-2)$, $y_1(3/2)$, and $y_1(2)$.

| | |
|------------|---|
| $Y_1(-2)$ | 0 |
| $Y_1(3/2)$ | 0 |
| $Y_1(2)$ | 0 |

Applications

Many applied problems can be solved by using equations such as those we have been solving.

EXAMPLE 5

Area of a room

Ronald's living room is 2 feet longer than it is wide, and its area is 168 square feet. What are the dimensions of the room?

Solution

Let x be the width and $x + 2$ be the length. See Figure 5.1. Because the area of a rectangle is the length times the width, we can write the equation

$$x(x + 2) = 168.$$

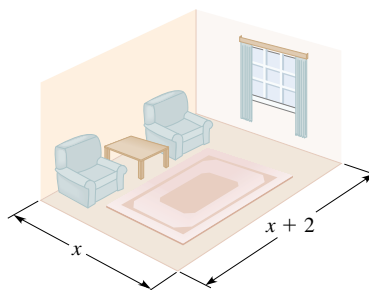


FIGURE 5.1

We solve the equation by factoring:

$$\begin{aligned} x^2 + 2x - 168 &= 0 \\ (x - 12)(x + 14) &= 0 \\ x - 12 = 0 &\quad \text{or} \quad x + 14 = 0 \\ x = 12 &\quad \text{or} \quad x = -14 \end{aligned}$$

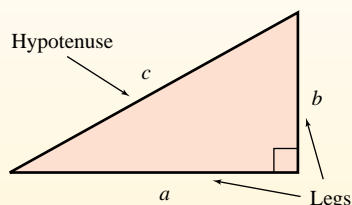
Because the width of a room is a positive number, we disregard the solution $x = -14$. We use $x = 12$ and get a width of 12 feet and a length of 14 feet. Check this answer by multiplying 12 and 14 to get 168. ■

Applications involving quadratic equations often require a theorem called the **Pythagorean theorem**. This theorem states that *in any right triangle the sum of the squares of the lengths of the legs is equal to the length of the hypotenuse squared*.

The Pythagorean Theorem

The triangle shown is a right triangle if and only if

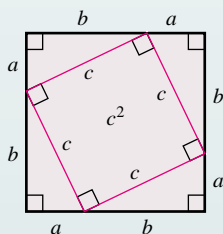
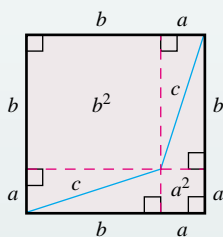
$$a^2 + b^2 = c^2.$$



We use the Pythagorean theorem in the next example.

helpful hint

To prove the Pythagorean theorem, draw two squares with sides of length $a + b$, and partition them as shown.

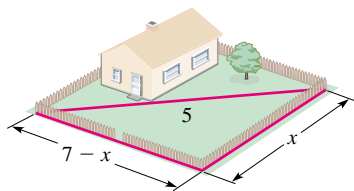


Erase the four triangles in each picture. Since we started with equal areas, we must have equal areas after erasing the triangles:

$$a^2 + b^2 = c^2$$

EXAMPLE 6 Using the Pythagorean theorem

Shirley used 14 meters of fencing to enclose a rectangular region. To be sure that the region was a rectangle, she measured the diagonals and found that they were 5 meters each. (If the opposite sides of a quadrilateral are equal and the diagonals are equal, then the quadrilateral is a rectangle.) What are the length and width of the rectangle?

**FIGURE 5.2****Solution**

The perimeter of a rectangle is twice the length plus twice the width, $P = 2L + 2W$. Because the perimeter is 14 meters, the sum of one length and one width is 7 meters. If we let x represent the width, then $7 - x$ is the length. We use the Pythagorean theorem to get a relationship among the length, width, and diagonal. See Figure 5.2.

$$\begin{aligned} x^2 + (7 - x)^2 &= 5^2 && \text{Pythagorean theorem} \\ x^2 + 49 - 14x + x^2 &= 25 && \text{Simplify.} \\ 2x^2 - 14x + 24 &= 0 && \text{Simplify.} \\ x^2 - 7x + 12 &= 0 && \text{Divide each side by 2.} \\ (x - 3)(x - 4) &= 0 && \text{Factor the left-hand side.} \\ x - 3 = 0 & \quad \text{or} \quad x - 4 = 0 && \text{Zero factor property} \\ x = 3 & \quad \text{or} \quad x = 4 \\ 7 - x = 4 & \quad \text{or} \quad 7 - x = 3 \end{aligned}$$

Solving the equation gives two possible rectangles: a 3 by 4 rectangle or a 4 by 3 rectangle. However, those are identical rectangles. The rectangle is 3 meters by 4 meters. ■

WARM - U P S**True or false? Explain your answer.**

- The equation $(x - 1)(x + 3) = 12$ is equivalent to $x - 1 = 3$ or $x + 3 = 4$.
False
- Equations solved by factoring may have two solutions. **True**
- The equation $c \cdot d = 0$ is equivalent to $c = 0$ or $d = 0$. **True**
- The equation $|x^2 + 4| = 5$ is equivalent to the compound equation $x^2 + 4 = 5$ or $x^2 - 4 = 5$. **False**
- The solution set to the equation $(2x - 1)(3x + 4) = 0$ is $\left\{\frac{1}{2}, -\frac{4}{3}\right\}$. **True**
- The Pythagorean theorem states that the sum of the squares of any two sides of any triangle is equal to the square of the third side. **False**
- If the perimeter of a rectangular room is 38 feet, then the sum of the length and width is 19 feet. **True**
- Two numbers that have a sum of 8 can be represented by x and $8 - x$.
True
- The solution set to the equation $x(x - 1)(x - 2) = 0$ is $\{1, 2\}$. **False**
- The solution set to the equation $3(x + 2)(x - 5) = 0$ is $\{3, -2, 5\}$.
False

5.9 EXERCISES

Reading and Writing After reading this section, write out the answers to these questions. Use complete sentences.

- What is the zero factor property?
The zero factor property says that if $a \cdot b = 0$ then either $a = 0$ or $b = 0$.
- What is a quadratic equation?
A quadratic equation is an equation of the form $ax^2 + bx + c = 0$ with $a \neq 0$.
- Where is the hypotenuse in a right triangle?
The hypotenuse of a right triangle is the side opposite the right angle.
- Where are the legs in a right triangle?
The legs of a right triangle are the sides that form the right angle.
- What is the Pythagorean theorem?
The Pythagorean theorem says that a triangle is a right triangle if and only if the sum of the squares of the legs is equal to the square of the hypotenuse.
- Where is the diagonal of a rectangle?
The diagonal of a rectangle is the line segment that joins two opposite vertices.

Solve each equation. See Examples 1–3.

- $(x - 5)(x + 4) = 0$ $\{-4, 5\}$
- $(a - 6)(a + 5) = 0$ $\{-5, 6\}$
- $(2x - 5)(3x + 4) = 0$ $\left\{\frac{5}{2}, -\frac{4}{3}\right\}$
- $(3k + 8)(4k - 3) = 0$ $\left\{-\frac{8}{3}, \frac{3}{4}\right\}$
- $w^2 + 5w - 14 = 0$ $\{-7, 2\}$
- $t^2 - 6t - 27 = 0$ $\{-3, 9\}$
- $m^2 - 7m = 0$ $\{0, 7\}$
- $h^2 - 5h = 0$ $\{0, 5\}$
- $a^2 - a = 20$ $\{-4, 5\}$
- $p^2 - p = 42$ $\{-6, 7\}$
- $3x^2 - 3x - 36 = 0$ $\{-3, 4\}$
- $-2x^2 - 16x - 24 = 0$ $\{-6, -2\}$
- $z^2 + \frac{3}{2}z = 10$ $\left\{-4, \frac{5}{2}\right\}$
- $m^2 + \frac{11}{3}m = -2$ $\left\{-3, -\frac{2}{3}\right\}$
- $x^3 - 4x = 0$ $\{-2, 0, 2\}$
- $16x - x^3 = 0$ $\{-4, 0, 4\}$
- $w^3 + 4w^2 - 25w - 100 = 0$ $\{-5, -4, 5\}$
- $a^3 + 2a^2 - 16a - 32 = 0$ $\{-4, -2, 4\}$
- $n^3 - 2n^2 - n + 2 = 0$ $\{-1, 1, 2\}$
- $w^3 - w^2 - 25w + 25 = 0$ $\{-5, 1, 5\}$

Solve each equation. See Example 4.

27. $|x^2 - 5| = 4$ $\{-3, -1, 1, 3\}$

- $|x^2 - 17| = 8$ $\{-5, -3, 3, 5\}$
- $|x^2 + 2x - 36| = 12$ $\{-8, -6, 4, 6\}$
- $|x^2 + 2x - 19| = 16$ $\{-7, -3, 1, 5\}$
- $|x^2 + 4x + 2| = 2$ $\{-4, -2, 0\}$
- $|x^2 + 8x + 8| = 8$ $\{-8, -4, 0\}$
- $|x^2 + 6x + 1| = 8$ $\{-7, -3, 1\}$
- $|x^2 - x - 21| = 9$ $\{-5, -3, 4, 6\}$

Solve each equation.

- $2x^2 - x = 6$ $\left\{-\frac{3}{2}, 2\right\}$
- $3x^2 + 14x = 5$ $\left\{-5, \frac{1}{3}\right\}$
- $|x^2 + 5x| = 6$ $\{-6, -3, -2, 1\}$
- $|x^2 + 6x - 4| = 12$ $\{-8, -4, -2, 2\}$
- $x^2 + 5x = 6$ $\{-6, 1\}$
- $x + 5x = 6$ $\{1\}$
- $(x + 2)(x + 1) = 12$ $\{-5, 2\}$
- $(x + 2)(x + 3) = 20$ $\{-7, 2\}$
- $y^3 + 9y^2 + 20y = 0$ $\{-5, -4, 0\}$
- $m^3 - 2m^2 - 3m = 0$ $\{-1, 0, 3\}$
- $5a^3 = 45a$ $\{-3, 0, 3\}$
- $5x^3 = 125x$ $\{-5, 0, 5\}$
- $(2x - 1)(x^2 - 9) = 0$ $\left\{-3, \frac{1}{2}, 3\right\}$
- $(x - 1)(x + 3)(x - 9) = 0$ $\{-3, 1, 9\}$
- $4x^2 - 12x + 9 = 0$ $\left\{\frac{3}{2}\right\}$

50. $16x^2 + 8x + 1 = 0$ $\left\{-\frac{1}{4}\right\}$

Solve each equation for y . Assume a and b are positive numbers.

- $y^2 + by = 0$ $\{0, -b\}$
- $y^2 + ay + by + ab = 0$ $\{-a, -b\}$
- $a^2y^2 - b^2 = 0$ $\left\{-\frac{b}{a}, \frac{b}{a}\right\}$
- $9y^2 + 6ay + a^2 = 0$ $\left\{-\frac{a}{3}\right\}$
- $4y^2 + 4by + b^2 = 0$ $\left\{-\frac{b}{2}\right\}$
- $y^2 - b^2 = 0$ $\{-b, b\}$
- $ay^2 + 3y - ay = 3$ $\left\{-\frac{3}{a}, 1\right\}$
- $a^2y^2 + 2aby + b^2 = 0$ $\left\{-\frac{b}{a}\right\}$

Solve each problem. See Examples 5 and 6.

59. **Color print.** The length of a new “super size” color print is 2 inches more than the width. If the area is 24 square inches, what are the length and width?

Width 4 inches, length 6 inches

60. **Tennis court dimensions.** In singles competition, each player plays on a rectangular area of 117 square yards. Given that the length of that area is 4 yards greater than its width, find the length and width.

Width 9 yards, length 13 yards

61. **Missing numbers.** The sum of two numbers is 13 and their product is 36. Find the numbers. 4 and 9

62. **More missing numbers.** The sum of two numbers is 6.5, and their product is 9. Find the numbers. 2 and 4.5

63. **Bodyboarding.** The Seamas Channel pro bodyboard shown in the figure has a length that is 21 inches greater than its width. Any rider weighing up to 200 pounds can use it because its surface area is 946 square inches. Find the length and width. Length 43 inches, width 22 inches

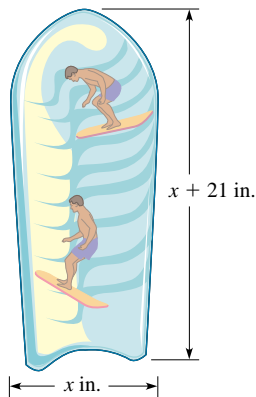


FIGURE FOR EXERCISE 63

64. **New dimensions in gardening.** Mary Gold has a rectangular flower bed that measures 4 feet by 6 feet. If she wants to increase the length and width by the same amount to have a flower bed of 48 square feet, then what will be the new dimensions? 6 feet by 8 feet

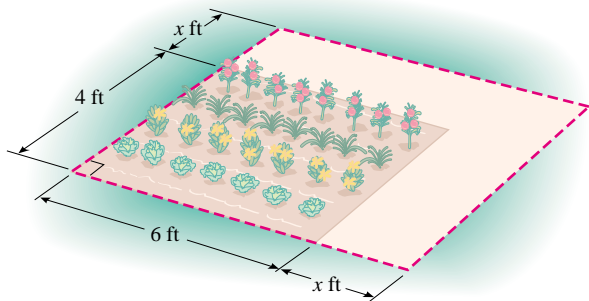


FIGURE FOR EXERCISE 64

65. **Shooting arrows.** An archer shoots an arrow straight upward at 64 feet per second. The height of the arrow $h(t)$ (in feet) at time t seconds is given by the function

$$h(t) = -16t^2 + 64t.$$

- a) Use the accompanying graph to estimate the amount of time that the arrow is in the air. 4 seconds
 b) Algebraically find the amount of time that the arrow is in the air. 4 seconds

- c) Use the accompanying graph to estimate the maximum height reached by the arrow. 64 feet
 d) At what time does the arrow reach its maximum height? 2 seconds

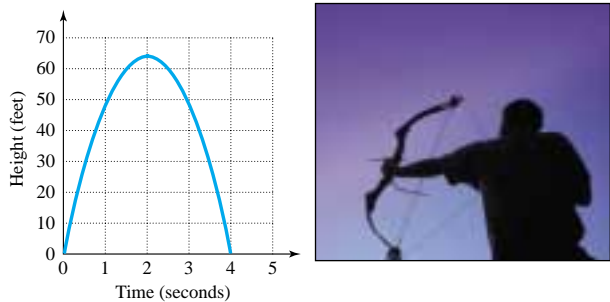


FIGURE FOR EXERCISE 65

66. **Time until impact.** If an object is dropped from a height of s_0 feet, then its altitude after t seconds is given by the formula $S = -16t^2 + s_0$. If a pack of emergency supplies is dropped from an airplane at a height of 1600 feet, then how long does it take for it to reach the ground? 10 seconds
 67. **Yolanda's closet.** The length of Yolanda's closet is 2 feet longer than twice its width. If the diagonal measures 13 feet, then what are the length and width?

Width 5 feet, length 12 feet

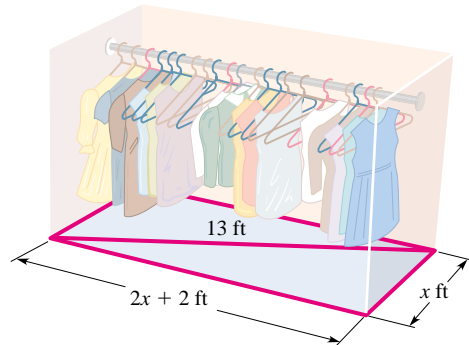


FIGURE FOR EXERCISE 67

68. **Ski jump.** The base of a ski ramp forms a right triangle. One leg of the triangle is 2 meters longer than the other. If the hypotenuse is 10 meters, then what are the lengths of the legs? 6 feet and 8 feet

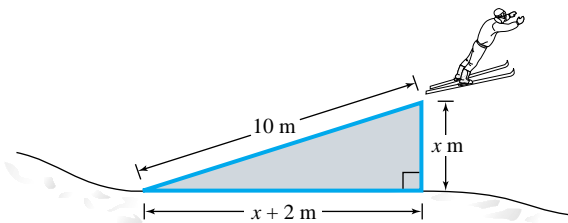


FIGURE FOR EXERCISE 68

69. **Trimming a gate.** A total of 34 feet of 1×4 lumber is used around the perimeter of the gate shown in the figure on the next page. If the diagonal brace is 13 feet long, then what are the length and width of the gate?
 Width 5 feet, length 12 feet

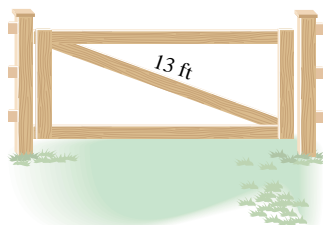


FIGURE FOR EXERCISE 69

70. **Perimeter of a rectangle.** The perimeter of a rectangle is 28 inches, and the diagonal measures 10 inches. What are the length and width of the rectangle?

Length 8 inches, width 6 inches

71. **Consecutive integers.** The sum of the squares of two consecutive integers is 25. Find the integers.

3 and 4, or -4 and -3

72. **Pete's garden.** Each row in Pete's garden is 3 feet wide. If the rows run north and south, he can have two more rows than if they run east and west. If the area of Pete's garden is 135 square feet, then what are the length and width?

Length 15 feet, width 9 feet

73. **House plans.** In the plans for their dream house the Baileys have a master bedroom that is 240 square feet in area. If they increase the width by 3 feet, they must decrease the length by 4 feet to keep the original area. What are the original dimensions of the bedroom?

Length 20 feet, width 12 feet

74. **Arranging the rows.** Mr. Converse has 112 students in his algebra class with an equal number in each row. If he arranges the desks so that he has one fewer rows, he will have two more students in each row. How many rows did he have originally? 8

GETTING MORE INVOLVED



75. **Writing.** If you divide each side of $x^2 = x$ by x , you get $x = 1$. If you subtract x from each side of $x^2 = x$, you get $x^2 - x = 0$, which has two solutions. Which method is correct? Explain.



76. **Cooperative learning.** Work with a group to examine the following solution to $x^2 - 2x = -1$:

$$x(x - 2) = -1$$

$$x = -1 \quad \text{or} \quad x - 2 = -1$$

$$x = -1 \quad \text{or} \quad x = 1$$

Is this method correct? Explain.



77. **Cooperative learning.** Work with a group to examine the following steps in the solution to $5x^2 - 5 = 0$

$$5(x^2 - 1) = 0$$

$$5(x - 1)(x + 1) = 0$$

$$x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = 1 \quad \text{or} \quad x = -1$$

What happened to the 5? Explain.

COLLABORATIVE ACTIVITIES

Magic Tricks

Jim and Sadar are talking one day after class.

Sadar: Jim, I have a trick for you. Think of a number between 1 and 10. I will ask you to do some things to this number. Then at the end tell me your result, and I will tell you your number.

Jim: Oh, yeah you probably rig it so the result is my number.

Sadar: Come on Jim, give it a try and see.

Jim: Okay, okay, I thought of a number.

Sadar: Good, now write it down, and don't let me see your paper. Now add x . Got that? Now multiply everything by 2.

Jim: Hey, I didn't know you were going to make me think! This is algebra!

Sadar: I know, now just do it. Okay, now square the polynomial. Got that? Now subtract $4x^2$.

Jim: How did you know I had a $4x^2$? I told you this was rigged!

Sadar: Of course it's rigged, or it wouldn't work. Do you want to finish or not?

Jim: Yeah, I guess so. Go ahead, what do I do next?

Sadar: Divide by 4. Okay, now subtract the x -term.

Jim: Just any old x -term? Got any particular coefficient in mind?

Grouping: Two students per group

Topic: Practice with exponent rules, multiplying polynomials

Sadar: Now stop teasing me. I know you only have one x -term left, so subtract it.

Jim: Ha, ha, I *could* give you a hint about the coefficient, but that wouldn't be fair, would it?

Sadar: Well you could, and then I could tell you your number, or you could just tell me the number you have left after subtracting.

Jim: Okay, the number I had left at the end was 25. Let's see if you can tell me what the coefficient of the x -term I subtracted is.

Sadar: Aha, then the number you chose at the beginning was 5, and the coefficient was 10!

Jim: Hey, you're right! How did you do that?

In your group, follow Sadar's instructions and determine why she knew Jim's number. Make up another set of instructions to use as a magic trick. Be sure to use variables and some of the exponent rules or rules for multiplying polynomials that you learned in this chapter. Exchange instructions with another group and see whether you can figure out how their trick works.